II - APPLICATION OF PROXIMITIES TO THERMODYNAMICS

II A) Homogeneity

$\mathcal{T}$ isan universal class of sets T

$\Pi_\gamma(T)$ is a partition of T

$T_\alpha, T_\beta \in \Pi_\gamma(T)$

φ is a set of linearly independant real measures (or σ-measures)

(1) ..... $\mu_i : T \longrightarrow R$ where: $\mu_i \in \varphi$, $T \in \mathcal{T}$ and R (Reals)

$\Pi_\gamma(T)$ being a partition of T, the following expressions apply:

(2) ..... $T_\alpha \cap T_\beta \equiv \phi \; , \forall \; (\alpha \neq \beta)$

(3) ..... $\bigcup\limits_\alpha (T_\alpha) = T$

DEF. 1 Partition $\Pi_\gamma(T)$ is φ-homogereous if:

(4) ..... $$\frac{\mu_i(T_\alpha)}{\mu_i(T_\beta)} = \frac{\mu_j(T_\alpha)}{\mu_j(T_\beta)} \; , \quad \begin{array}{l} \forall \; T_\alpha, T_\beta \in \Pi_\gamma(T) \\ \forall \; \mu_i, \mu_j \in \varphi \end{array}$$ and

expression (4) can be easily transformed in (5):

(5) $$\frac{\mu_i(T_\alpha)}{\mu_i(T)} = \frac{\mu_j(T_\alpha)}{\mu_j(T)} \; , \quad \begin{array}{l} \forall \; T_\alpha \in \Pi_\gamma(T) \\ \forall \; \mu_i, \mu_j \in \varphi \end{array}$$

DEF. 2 The fineness ρ of a partition $\Pi_\gamma(T)$ is defined by (6):

(6) $$\mathrm{Max}\left[\frac{\mu_i(T_\alpha)}{\mu_i(T)} \; , \; \forall \; T_\alpha \in \Pi_\gamma(T) \right] = \rho_{\Pi_\gamma}(T)$$

Taking in consideration (4) and (5), $\rho_{\Pi_\gamma}(T)$ is independant of μ_i.

DEF. 3 : $\Pi_T \equiv \left\{ \Pi_\gamma(T) : \rho_{\Pi_\gamma(T)} \leq \rho \right\}$ ..... (7)

is the set of all partitions of T that have finenesses less than ρ .

Π_T is $\rho\varphi$ -homogeneous .

The set T possessing a non void set of partitions $\rho\varphi$ -homogeneous is defined as $\rho\varphi$ -homogeneous.

DEF. 4; $\mathcal{T}^\times \equiv \left\{ T : T \in \mathcal{T} \text{ and } T \ \rho\varphi\text{-homogeneous} \right\}$

$\mathcal{T}^\times$ is a sub-set of $\mathcal{T}$ and is considered $\rho\varphi$ -homogeneous.

DEF. 5: If:

$$\forall \mu_i \in \varphi , \ \mu_i(T) = \mu_i(T') \Longleftrightarrow T \equiv T'$$

$$\mu_i \in \varphi , \ \mu_i(T) \neq \mu_i(T') \Longleftrightarrow T \not\equiv T'$$

for $\forall$, T , $T' \in \mathcal{T}^\times \subseteq \mathcal{T}$ and $\rho\varphi$ -homogeneous.

Then φ is an "adequate" set of linearly independant real measures (σ -measures) for $\mathcal{T}^\times$.

Note that any other real measures on T can be expressed as a linear homogeneous function of degree 1 of the measures belonging to φ .

II B) An Axiomatic for Thermostatic

Ax. 1 : All thermodynamic system is an universal class of sets $\mathcal{T}^\times$, $\rho\varphi$ -homogeneous and φ is an "adequate" set of linearly independant real measures (σ -measures) for $\mathcal{T}^\times$.

Card $\varphi = N$, finite.

For $\rho \longrightarrow 0$ (zero), $\forall \mu_i \in \varphi$, $\mu_i(T)$ is continuous on $\mathcal{T}^\times$.

Ax. 2 : There are two real measures (σ -measures), entropy μ_s and internal energy μ_u .

If $\varphi \equiv \left\{ \mu_s ; \mu_i \dots \mu_j \right\}$ then $\mu_u = F[\varphi]$ and $\varphi \cup \mu_u$ is a N + 1 Euclidean Convex Space.

μ_u and μ_s are dual functions, expratis:

$$(\min \mu_u)_{\mu_s = \text{const.}} = (\max \mu_s)_{\mu_u = \text{const.}}$$

Note 1 : Continuous trajectories (lines) can be described on the surface

$$\mu_\mu - F\left[\mu_s, \mu_i \dots, \mu_j\right] = 0 \text{ if } p \to 0.$$

Note 2 : The thermodynamic space is not metrisable, but a proximity can be defined, as it will be shown in II C).

II C) Proximity in thermodynamics

1) Reversible and irreversible trajectories

In all trajectories (reversible or otherwise) the starting state T^x and finishing state T^y belong to $\mathcal{G}^x$ $p\varphi$ -homogeneous.

If all the other intermediate states belong to $\mathcal{G}^x$ then the trajectory is declared reversible, if not irreversible.

A general irreversible trajectory is symbolised in the following fashion:

$T^x \nearrow\!\!\searrow T^y$, T^x, T^y being respectively the starting and finishing point and $T^x, T^y \in \mathcal{G}^x$.

Reversible trajectories are represented as follows: $T^x \longrightarrow T^y$

2) Definition of a Proximity on $\mathcal{G}^x$

$$\mathcal{H} = \bigtimes_i \mu_i \;, \forall \mu_i \in \varphi \qquad \text{(Cartesean Product)}$$

The proximity of two states T^x T^y is given by:

$$\mathcal{H}(x,y) = \Gamma_{xy}(\alpha)$$

where $\Gamma_{x,y}(\alpha) \in \mathcal{C}^T \subseteq \mathcal{C}$ (defined in 2 e)

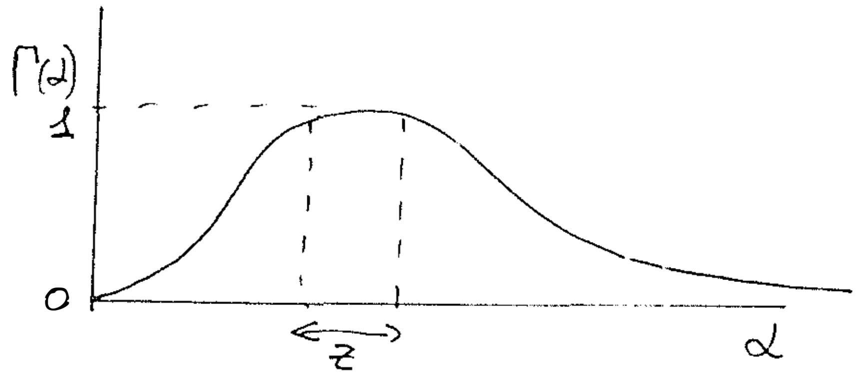
$\mathcal{C}^T$ satisfies to the following conditions:

a) $\mathcal{C}^T \in \mathcal{C}$

b) $\Gamma(0) = 0$

c) The non-decreasing branch starts at $\alpha = 0$.

Thus the general aspect of Γ is the following:



z corresponds to the region of the "real" trajectories, the most plausible, and the reversible trajectories to $\alpha = 0$, $\Gamma(0) = 0$, which can be interpreted as "impractible" because $\Gamma(0) = 0$.

Comments:

1) All reversible trajectories are equiproximate $\alpha = 0$, and their likelihood, $\Gamma(\alpha = 0) = 0$, is zero, physically ideal.
2) All irreversible trajectories correspond to $\alpha > 0$ and their likelihood is positive $\Gamma(\alpha > 0) > 0$.
3) The most likely proximity corresponds to the zone where $\Gamma(\alpha)$ is maximum.
4) Considering the two trajectories, $T^x \nearrow\!\searrow T^y \nearrow\!\searrow T^z \Longleftrightarrow \Gamma_{xyz}(\gamma)$ and $T^x \nearrow\!\searrow T^z \Longleftrightarrow \Gamma_{xz}(\gamma)$. The zone of higher likelihood is shifted to greater values of α in Γ_{xyz} than in Γ_{xz} .

II D) Entropy Production

If α is interpreted as entropy production, $\alpha \equiv \delta S$, then $\Gamma_{xy}(\alpha) \equiv \Gamma_{xy}(\delta S)$ and some simple conclusions can be taken:

- In a reversible trajectory (process), $\alpha = 0$ then $\Gamma_{xy}(0) = 0$, the likelihood of such a process is nill.
- The max $\left[\Gamma_{xy}(\alpha)\right] = \Gamma^*$ corresponds to the entropy production $\delta S \equiv \alpha$ more likely to occur.
- The set $\left\{\alpha : \Gamma_{xy}(\alpha) \geqslant \delta \leqslant \Gamma^*\right\} \equiv [\delta_a, \delta_b]$ is an interval of occurence of trajectories which are δ-likely to occur.

- If, $\max\left[\Gamma_{xyz}(\gamma)\right] = \Gamma^{x}_{xyz}$ and $\max\left[\Gamma_{xz}(\alpha)\right] = \Gamma^{x}_{xz}$

and
$$[\gamma^{x}_{a}, \gamma^{x}_{b}] \equiv \left\{\gamma : \Gamma_{xyz}(\gamma) = \Gamma^{x}_{xyz}\right\}$$
$$[\delta^{x}_{a}, \delta_{b}] \equiv \left\{\alpha : \Gamma_{xz}(\alpha) = \Gamma^{x}_{xz}\right\}$$

then $\Gamma^{x}_{xz} \geqslant \Gamma^{x}_{xyz}$

and $\delta^{x}_{a} \leq \gamma^{x}_{a}$ and $\delta^{x}_{b} \leqslant \gamma^{x}_{b}$

which means the entropy production in the process $T^{x} \curvearrowright T^{y} \curvearrowright T^{z}$ is greater than in the process $T^{x} \curvearrowright T^{z}$, for the same likelihood (or level of membership).

II E) Time and Entropy Production

If time t is considered a monotonous function of $1/\alpha \equiv 1/\delta S$, some interesting interpretations are possible.

a) If $\alpha = 0$ then $t = \infty$. A process that would take ∞ time for completion would be eventually reversible.

b) The typical t^{x} (or the most likely time) would correspond to $\Gamma^{x}\left(\max \Gamma(\alpha)\right)$.

c) As $\alpha \rightarrow \infty$, $t \rightarrow 0$, and $\Gamma(\alpha) \rightarrow 0$. this could be interpreted as follows: when the time of the process is less than t^{x}, then the likelihood of the process would diminish tending to zero with $t \rightarrow 0$.

Conclusion

Space $\mathfrak{X} \equiv \varphi$ can be topologically structured with a class $\beta^{x} \leqslant \beta$ of proximities and some form of a fuzzy distance, Proximity, between thermodynamic states can be defined.

Entropy production is a monotonous function of α, eventually $\alpha \equiv \delta S$.

Time is an inverse function of δS.