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INTRODUCTION

The paper presents a certain class of "proximities" suitable to thermodynamics and mechanics.

Proximity or Prox was introduced in 1974, see references (1), (2) and (3), and some classes of Prox were built using the theory of fuzzy mathematics, see reference (4).

Regarding fuzzy mathematics, some typical references were chosen, as for instance (5), (6), (7), (8), (9), (10), (11), (12), (13) and (14).

A summary of applications of the concept of Proximity was presented to the "1º Congrès de H.B.D.S.", held in Lisbon, ref. (15).

The Part I is an introduction to the concept of proximity taking values on a Fuzzy Topological Space of a certain kind.

In Part II, an application of the concept of proximity is made to thermodynamics.

PART I

Definition: PROX (x,y) is a function $\mathfrak{R}(x,y)$, defined on $\mathcal{X}^2$ and taking values in $\mathcal{G}$.

Function $\mathfrak{R}(x,y) : \mathcal{X}^2 \longrightarrow \mathcal{G}$

When: $\mathcal{X}$ is a set, whose elements are eventually sets

$\mathcal{G} \subseteq \Omega^{R+}$ and $\Gamma(\alpha) \in \Omega$, $\forall \alpha \in R^+$,

Ω is a net $(\Omega, \Psi, \pitchfork)$

Ψ and $\pitchfork$ are the two laws of internal composition defined on Ω (Ex.gr. MAX-MIN or MAJ-MIN)

$\succcurlyeq$ is the symbol of the partial order induced in the set Ω.

1 is the supremum

0 is the infimum

R^+ symbolises the non negative real numbers

Finally Ω is completely distributive, and an involutive operator (,) is defined such that the order $\succcurlyeq$ is inverted.

The principal properties of $\mathcal{G}$ are:

1,a) $\mathcal{G} \subseteq \Omega^{R^+}$ (by definition)

1,b) If $\Gamma^* = \Psi\left[\Gamma(\alpha), \forall \alpha \in R^+\right]$ and $\{\alpha^*\} \equiv \{\alpha : \Gamma(\alpha) = \Gamma^*\}$

then: $\{\alpha^*\} \equiv [\alpha_a^*, \alpha_b^*] \subseteq R^+$ (see Annex A)

eventually $\{\alpha^*\}$ is a singleton.

1,c) $\Gamma(\alpha)$ is a monotonously non decreasing,

$\forall \alpha \in [0, \alpha_a^*[$, $\alpha_0^* \geqslant 0$

1,d) $\Gamma(\alpha)$ is monotonously non increasing $\forall \alpha \in]\alpha_b, \infty[$

1,e) $\Gamma(\alpha) \longrightarrow 0$, $\alpha \longrightarrow \infty$

1,f) $\Gamma(0) = 0$

1,g) $\Gamma(\alpha)$ can have numerable discontinuities.

The fundamental properties imposed on $\mathfrak{R}(x,y)$ are:

2,a) $\left[\mathfrak{R}(x,y) = \mathfrak{R}(y,x)\right] \Longleftrightarrow \left[\Gamma_{x,y}(\alpha) = \Gamma_{yx}(\alpha)\right]$

(commutative) $\forall, \alpha \in R^+$

2,b) Composition $\oplus$

$\mathfrak{R}(x,y) \oplus \mathfrak{R}(y,z)$ is defined as:

$$\Psi\left\{ \pitchfork \left[\Gamma_{xy}(\alpha), \Gamma_{yz}(\beta) \right] \right\} = \Gamma_{x,y,z}(\gamma)$$

where $\gamma = \alpha + \beta$ and $\alpha, \beta, \gamma \in R^{+}$

The composition operator is:

- closed and commutative (see Annex B)

2,c) $\mathcal{R}(x,x) \longrightarrow \Gamma_{xx}(\alpha) = 0$ (infimum) $, \forall \alpha \in R^{+}, \forall x \in \mathfrak{X}$

2,d) If: $\Gamma^{*}_{xyz} \equiv [\gamma^{*}_{a}, \gamma^{*}_{b}]$ and $\Gamma^{*}_{xz} \equiv [\delta^{*}_{o}, \delta^{*}_{b}]$

then $\Gamma^{*}_{xz} \geqslant \Gamma^{*}_{xyz}$

and $\delta^{*}_{a} \leqslant \gamma^{*}_{a}$ and $\delta^{*}_{b} \leqslant \gamma^{*}_{b}$

This last property can be viewed as a fuzzy triangular inequality.

Note that, if crisp sets and crisp triangular law is used than a normal "écart" is obtained.

2,e) Proximities satisfying the above mentioned properties are denominated $\mathcal{G}$-class proximities.

ANNEX A :

1) Definition: Crisp set A with membership index ε, is the set.

$$A_{\varepsilon} \equiv \left\{ \alpha : \Gamma(\alpha) \geqslant \varepsilon , \varepsilon \in \Omega \right\}$$

2) If $\varepsilon^{*} \equiv \Upsilon\left[\Gamma(\alpha), \forall \alpha \in R^{+} \right]$

then A^{*}_{ε} is the interval refered in 1,b).

3) It is easily seen that if $\varepsilon > \varepsilon^{*}$ then $A_{\varepsilon} \equiv \phi$

4) If $\Gamma(\alpha) = \Gamma^{*} = 1$ for $\alpha = \alpha_{o}$

$\Gamma(\alpha) = 0$ for $\forall \alpha \neq \alpha_{o}$ and $\alpha \in R^{+}$

then a fuzzy set is reduced to a crisp set.

ANNEX B :

Some prooves are presented here to show that the composition $\mathcal{R}(x,y) \oplus \mathcal{R}(y,z)$ produces a function $\Gamma_{xyz}(\gamma)$ of the same class of $\Gamma_{xy}(\alpha)$ and $\Gamma_{yz}(\beta)$, ex. gr. : Class $\mathcal{G}$.

- Proof of 1,b)

Let be given the function $\Gamma_{xy}(\alpha)$ and $\Gamma_{yz}(\beta)$ satisfying the properties 1,a) to 1,g).

The following symbols are introduced:

$$\Gamma_{xy}^{*}(\alpha) = \Psi\left[\Gamma_{xy}(\alpha),\ \forall\alpha\in R^{+}\right]$$

$$\Gamma_{yz}^{*}(\beta) = \Psi\left[\Gamma_{xy}(\beta),\ \forall\beta\in R^{+}\right]$$

$$\varepsilon = \pitchfork\left[\Gamma_{xy}^{*},\ \Gamma_{yz}^{*}\right]$$

$$[\alpha_a, \alpha_b]_\varepsilon = \{\alpha : \Gamma_{xy}(\alpha) \geqslant \varepsilon\}$$

$$[\beta_c, \beta_d]_\varepsilon = \{\beta : \Gamma_{yz}(\beta) \geqslant \varepsilon\}$$

If

$$(\alpha\in[\alpha_a,\alpha_b]_\varepsilon)\wedge(\beta\in[\beta_c,\beta_d]_\varepsilon)$$

then:

$$\Gamma_{xyz}(\gamma) = \varepsilon$$

$$\Gamma_{xyz}^{*} = \varepsilon$$

$$[\gamma_e^{*}, \gamma_f^{*}] \equiv \{\gamma : \Gamma_{xyz}(\gamma) = \Gamma_{xyz}^{*} = \varepsilon\}$$

where:

$$\Gamma_{xyz}(\gamma) = \Gamma_{xy}(\alpha) \oplus \Gamma_{yz}(\beta)$$

$$\gamma = \alpha + \beta$$

$$\Gamma_{xyz}^{*} = \Psi\left[\Gamma_{xyz}(\gamma) : \forall\gamma\in R^{+}\right]$$

$$\gamma_e^{*} = \alpha_a + \beta_c$$

$$\gamma_f^{*} = \alpha_b + \beta_d$$

Finally it is important to note that:

or $[\alpha_a, \alpha_b]_\varepsilon \equiv [\alpha_a^{*}, \alpha_b^{*}]$

or $[\beta_c, \beta_d]_\varepsilon \equiv [\beta_c^{*}, \beta_d^{*}]$

or both.

- Proof of 1,c)

$\gamma \leqslant \gamma_e^{*}$ is a monotonously non decreasing branch.

$$\gamma_e^{*} = \alpha_a + \beta_c$$

$$\begin{cases} \Gamma_{xy}(\alpha_2) \geqslant \Gamma_{xy}(\alpha_1) & \forall\ \alpha_2 \geqslant \alpha_1 \\ \Gamma_{yz}(\beta_2) \geqslant \Gamma_{yz}(\beta_1) & \forall\ \beta_2 \geqslant \beta \end{cases} \quad \text{and} \quad \alpha_2+\beta_2 = \alpha_1+\beta_1 = \gamma \leqslant \gamma_e^{*}$$

both branches being monotonously non decreasing.

Then:

$$\Psi\left\{ [\ (\alpha_2),\ \Gamma_{yz}(\beta_2)] \right\} \geq$$

$$\Psi\left\{ [\ (\alpha_1),\ \Gamma_{yz}(\beta_1)] \right\}$$

or

$$(\gamma_2) \geq \Gamma_{xyz}(\gamma_1),\ \forall\ \gamma_2 \geq \gamma_1 \text{ and } \gamma_2 \leq \gamma_1^x$$

- Proof of [illegible]

The same [illegible] of demonstration as for 1,c) is appliable.

- Proof of [illegible]

It is str[illegible]forward, as $\gamma = \alpha + \beta$

$\Gamma(\gamma)$ when $\gamma \longrightarrow \infty$

- Proof of [illegible] As $\alpha, \beta, \gamma \geqslant 0$ then $\gamma = 0 \Longrightarrow (\alpha \text{ and } \beta = 0) \Rightarrow \Gamma_{xyz}(\gamma) = 0$

- Proof of [illegible]

Both Γ_{xy} and $\Gamma_{yz}(\beta)$ can have numerable discontinuities, this imp[illegible] their composition $\Gamma_{xy}(\alpha) \oplus \Gamma_{yz}(\beta) = \Gamma_{xyz}(\gamma)$

can have [illegible], numerable discontinuities.

- Proof of [illegible]

It is cl[illegible] $\Gamma_{xyz}(\gamma) < \Omega^R$

Note that $(\gamma) = 0$ for $x = y = z$ which is similar to

$\Gamma_{xy}(\alpha) = 0$ for $x = y$, but Γ_{xyz} may be different from zero

if $y \neq x$ $\Gamma_{xy} \equiv 0$

This concl[illegible] proof that, if Γ_{xy}, $\Gamma_{yz} \in \mathcal{C}$ than $\Gamma_{xyz} \in \mathcal{C}$.

ANNEX C

Definition fuzzy sphere

A fuzzy sp[illegible]entered in x, with radius r and with a degree of fuzziness, [illegible] is the set of points defined by the following expression:

$$B_x \equiv \left\{ \overset{r}{\underset{\varepsilon}{x}}(\gamma) \right\} \equiv \left\{ y : \Gamma_{xy}(\alpha) \succcurlyeq \varepsilon, [\alpha_a, \alpha_b] \leq [0^+, r] \right\}$$

where: $x, y \in \mathcal{H}$

$\varepsilon \in \Omega$

$\succcurlyeq$ order relation of the net Ω

$\mathcal{A}(x,y) \longrightarrow \Gamma_{xy}(\alpha) \in G \leq \Omega^R$

$[\alpha_a, \alpha_b]_\varepsilon \longrightarrow \left\{ \alpha : \Gamma_{xy}(\alpha) \succcurlyeq \varepsilon \text{ and } \alpha \in R^+ \right\}$

Fuzzy spheres depend on two parameters, namely radius r and degree of fuzziness (or membership) ε:

The less fuzzy sphere corresponds to $\varepsilon = \Gamma^x_{xy}$, ~~when~~

$$\Gamma^x_{xy} \equiv \Psi[\Gamma_{xy}(\alpha), \forall \alpha \in R^+]$$

If $\varepsilon > \Gamma^x_{xy}$ the sphere is empty, no points satisfying the definition.

Fuzzy-Topology of $\mathcal{H}$

Two spheres are given:

$$B_y \equiv \left\{ \overset{r}{\underset{\varepsilon_y}{y}}(x) \right\} \qquad \text{and} \qquad B_z \equiv \left\{ \overset{s}{\underset{\varepsilon_x}{z}}(x) \right\}$$

and $A \equiv \left\{ x : x \in B_y \cap B_z \right\} \not\equiv \phi$

$$\varepsilon^x \equiv \uparrow [\Gamma^x_{yx}, \Gamma^x_{zx}] \quad \text{and} \quad \varepsilon < \varepsilon^x$$

Defining a third sphere $B_x \equiv \left\{ \overset{t}{\underset{\varepsilon}{x}}(\omega) \right\}$

where: $x \in$ interior of A

$\omega \in \left\{ v : \Gamma_{xv}(\gamma) \succcurlyeq \varepsilon \text{ and } [\gamma_e, \gamma_f]_\varepsilon \leq [0, t] \right\}$

$t = \min[(r - \alpha_2), (s - \beta_\varepsilon)]$

$$\begin{cases} \alpha_\varepsilon \in]\alpha_1, \alpha_2[\; \subset [\alpha_a, \alpha_b]_\varepsilon \\ \beta_\varepsilon \in]\beta_1, \beta_2[\; \subset [\beta_c, \beta_d]_\varepsilon \end{cases}$$

These last two expressions can be stated as:

$$\alpha_\varepsilon \in INT[\alpha_a, \alpha_b]_\varepsilon$$

$$\beta_\varepsilon \in INT[\beta_c, \beta_d]_\varepsilon$$

Taking in consideration the fuzzy inequality law imposed on $\mathcal{C}$ -Prox, the sphere B_x has its points in the interior of $B_y \cap B_z$.
A family of spheres (open spheres) can be a base for fuzzy topology $\mathcal{P}$ for $\mathcal{X}$ and a fuzzy topological space $(\mathcal{H}, \mathcal{P})$ is formed.

There are other means to create a topology for $\mathcal{H}$, ex-gratis:

- A topology $\mathcal{T}$ can be imposed on Ω^2 , see references.
- From $\mathcal{T}$ can be defined a quocient-topology $\mathcal{T}_{\mathcal{C}}$ on $\mathcal{C}$.
- By means of a function Υ transfer $\mathcal{T}_{\mathcal{C}}$ to $\mathcal{H}^2$, where $\Upsilon \equiv A(x,y) \rightarrow$ $\rightarrow (\Gamma_{xy}(\alpha))$, and θ is the topology on $\mathcal{H}^2$.
- In each section of $\mathcal{H}^2$ by a plane $x = a$, the topology θ is induced. Let us call it θ_a.

  Of course, θ_a and $\mathcal{P}$ are different , in general.

II - APPLICATION OF PROXIMITIES TO THERMODYNAMICS

II A) Homogeneity

$\mathcal{T}$ isan universal class of sets T

$\Pi_\gamma(T)$ is a partition of T

$T_\alpha, T_\beta \in \Pi_\gamma(T)$

φ is a set of linearly independant real measures (or σ-measures)

(1) ..... $\mu_i : T \rightarrow R$ where: $\mu_i \in \varphi$, $T \in \mathcal{T}$ and R (Reals)

$\Pi_\gamma(T)$ being a partition of T, the following expressions apply:

(2) ..... $T_\alpha \cap T_\beta \equiv \phi$, $\forall (\alpha \neq \beta)$

(3) ..... $\bigcup\limits_\alpha (T_\alpha) = T$

DEF. 1 Partition $\Pi_\gamma(T)$ is φ-homogeneous if:

(4) ..... $$\frac{\mu_i(T_\alpha)}{\mu_i(T_\beta)} = \frac{\mu_j(T_\alpha)}{\mu_j(T_\beta)}, \quad \begin{array}{l} \forall\, T_\alpha, T_\beta \in \Pi_\gamma(T) \\ \forall\, \mu_i, \mu_j \in \varphi \end{array} \quad \text{and}$$

expression (4) can be easily transformed in (5);

(5) $$\frac{\mu_i(T_\alpha)}{\mu_i(T)} = \frac{\mu_j(T_\alpha)}{\mu_j(T)}, \quad \begin{array}{l} \forall\, T_\alpha \in \Pi_\gamma(T) \\ \forall\, \mu_i, \mu_j \in \varphi \end{array}$$

DEF. 2 The fineness ρ of a partition $\Pi_\gamma(T)$ is defined by (6):

(6) $$\mathrm{Max}\left[\frac{\mu_i(T_\alpha)}{\mu_i(T)}, \ \forall\, T_\alpha \in \Pi_\gamma(T)\right] = \rho_{\Pi_\gamma}(T)$$

Taking in consideration (4) and (5), $\rho_{\Pi_\gamma}(T)$ is independant of μ_i.

DEF. 3 : $\Pi_T \equiv \left\{ \Pi_\gamma(T) : \rho_{\Pi_\gamma(T)} \leq \rho \right\}$ . . . . . (7)

is the set of all partitions of T that have finenesses less than ρ .

Π_T is $\rho\varphi$-homogeneous.

The set T possessing a non void set of partitions $\rho\varphi$-homogeneous is defined as $\rho\varphi$-homogeneous.

DEF. 4: $\mathcal{T}^* \equiv \left\{ T : T \in \mathcal{T} \text{ and } T \; \rho\varphi\text{-homogeneous} \right\}$

$\mathcal{T}^*$ is a sub-set of $\mathcal{T}$ and is considered $\rho\varphi$-homogeneous.

DEF. 5: If:

$$\forall \mu_i \in \varphi \,,\; \mu_i(T) = \mu_i(T') \Longleftrightarrow T \equiv T'$$

$$\mu_i \in \varphi \,,\; \mu_i(T) \neq \mu_i(T') \Longleftrightarrow T \not\equiv T'$$

for $\forall$, T , $T' \in \mathcal{T}^* \subseteq \mathcal{T}$ and $\rho\varphi$-homogeneous.

Then φ is an "adequate" set of linearly independant real measures (σ -measures) for $\mathcal{T}^*$.

Note that any other real measures on T can be expressed as a linear homogeneous function of degree 1 of the measures belonging to φ .

II E) An Axiomatic for Thermostatic

Ax. 1 : All thermodynamic system is an universal class of sets $\mathcal{T}^*$, $\rho\varphi$-homogeneous and φ is an "adequate" set of linearly independant real measures (σ -measures) for $\mathcal{T}^*$.

Card $\varphi = N$, finite.

For $\rho \longrightarrow 0$ (zero), $\forall \mu_i \in \varphi$, $\mu_i(T)$ is continuous on $\mathcal{T}^*$.

Ax. 2 : There are two real measures (σ -measures), entropy μ_s and internal energy μ_u .

If $\varphi \equiv \left\{ \mu_s ; \mu_i \cdots \mu_j \right\}$ then $\mu_u = F[\varphi]$ and $\varphi \cup \mu_u$ is a N + 1 Euclidean Convex Space.

μ_u and μ_s are dual functions, exgratis:

$$(\min \mu_u)_{\mu_s = \text{const.}} = (\max \mu_s)_{\mu_u = \text{const.}}$$

Note 1 : Continuous trajectories (lines) can be described on the surface

$$\mu_{\mu} - F\left[\mu_s , \mu_i \ldots , \mu_j\right] = 0 \text{ if } \wp \to 0.$$

Note 2 : The thermodynamic space is not metrisable, but a proximity can be defined, as it will be shown in II C).

II C) Proximity in thermodynamics

1) Reversible and irreversible trajectories

In all trajectories (reversible or otherwise) the starting state T^x and finishing state T^y belong to $\mathcal{G}^x$ $\wp\varphi$ -homogeneous.

If all the other intermediate states belong to $\mathcal{G}^x$ then the trajectory is declared reversible, if not irreversible.

A general irreversible trajectory is symbolised in the following fashion:

$T^x \nearrow\!\!\searrow T^y$, T^x, T^y being respectively the starting and finishing point and $T^x, T^y \in \mathcal{G}^x$.

Reversible trajectories are represented as follows: $T^x \longrightarrow T^y$

2) Definition of a Proximity on $\mathcal{G}^x$

$$\mathcal{H} = \bigtimes_i \mu_i \, , \forall \mu_i \in \varphi \qquad \text{(Cartesean Product)}$$

The proximity of two states T^x T^y is given by:

$$\mathcal{R}(x,y) = \Gamma_{xy}(\alpha)$$

where $\Gamma_{x,y}(\alpha) \in \mathcal{C}^T \subseteq \mathcal{C}$ (defined in 2 e)

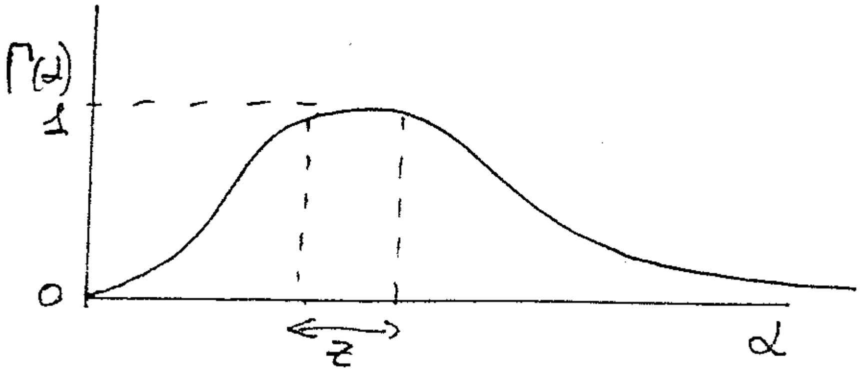
$\mathcal{C}^T$ satisfies to the following conditions:

a) $\mathcal{C}^T \in \mathcal{C}$

b) $\Gamma(0) = 0$

c) The non-decreasing branch starts at $\alpha = 0$.

Thus the general aspect of Γ is the following:



z corresponds to the region of the "real" trajectories, the most plausible, and the reversible trajectories to $\alpha = 0$, $\Gamma(0) = 0$, which can be interpreted as "impractible" because $\Gamma(0) = 0$.

Comments:

1) All reversible trajectories are equiproximate $\alpha = 0$, and their likelihood, $\Gamma(\alpha = 0) = 0$, is zero, physically ideal.

2) All irreversible trajectories correspond to $\alpha > 0$ and their likelihood is positive $\Gamma(\alpha > 0) > 0$.

3) The most likely proximity corresponds to the zone where $\Gamma(\alpha)$ is maximum.

4) Considering the two trajectories, $T^x \curvearrowright T^y \curvearrowright T^z \Longleftrightarrow \Gamma_{xyz}(\tau)$ and $T^x \curvearrowright T^z \Longleftrightarrow \Gamma_{xz}(\tau)$. The zone of higher likelihood is shifted to greater values of α in Γ_{xyz} than in Γ_{xz}.

II 0) Entropy Production

If α is interpreted as entropy production, $\alpha \equiv \delta S$, then $\Gamma_{xy}(\alpha) \equiv \Gamma_{xy}(\delta S)$ and some simple conclusions can be taken:

- In a reversible trajectory (process), $\alpha = 0$ then $\Gamma_{xy}(0) = 0$, the likelihood of such a process is nill.
- The max $\left[\Gamma_{xy}(\alpha)\right] = \Gamma^*$ corresponds to the entropy production $\delta S = \alpha$ more likely to occur.
- The set $\left\{\alpha : \Gamma_{xy}(\alpha) \geqslant \delta \leqslant \Gamma^*\right\} \equiv \left[\delta_a, \delta_b\right]$ is an interval of occurence of trajectories which are δ-likely to occur.

- If, max $\left[\Gamma_{xyz}(\gamma)\right] = \Gamma^*_{xyz}$ and $\max\left[\Gamma_{xz}(\alpha)\right] = \Gamma^*_{xz}$

and
$$[\gamma_a^*, \gamma_b^*] \equiv \{\gamma : \Gamma_{xyz}(\gamma) = \Gamma^*_{xyz}\}$$
$$[\delta_a^*, \delta_b^*] \equiv \{\alpha : \Gamma_{xz}(\alpha) = \Gamma^*_{xz}\}$$

then $\Gamma^*_{xz} \geqslant \Gamma^*_{xyz}$

and $\delta_a^* \leq \gamma_a^*$ and $\delta_b^* \leqslant \gamma_b^*$

which means the entropy production in the process $T^x \searrow T^y \searrow T^z$ is greater than in the process $T^x \searrow T^z$, for the same likelihood (or level of membership).

II E) Time and Entropy Production

If time t is considered a monotonous function of $1/\alpha \equiv 1/\delta S$, some interesting interpretations are possible.

a) If $\alpha = 0$ then $t = \infty$. A process that would take ∞ time for completion would be eventually reversible.

b) The typical t^* (or the most likely time) would correspond to $\Gamma^*(\max\Gamma(\alpha))$.

c) As $\alpha \to \infty$, $t \to 0$, and $\Gamma(\alpha) \to 0$. this could be interpreted as follows: when the time of the process is less than t^*, then the likelihood of the process would diminish tending to zero with $t \to 0$.

Conclusion

Space $\mathfrak{X} \equiv \varphi$ can be topologically structured with a class $\mathcal{C}^* \leqslant \mathcal{C}$ of proximities and some form of a fuzzy distance, Proximity, between thermodynamic states can be defined.

Entropy production is a monotonous function of α, eventually $\alpha \equiv \delta S$.

Time is an inverse function of δS.

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