AN AXIOMATIC ON PROXIMITIES

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Summary: An introduction of the concept of Proximity, namely $\mathcal{C}$-proximities, is given, a typical axiomatic is presented, and an application to thermodynamics shows that $\mathcal{C}$-proximities are rather flexible. This is clearly a preliminar presentation but we hope to have shown the power of fuzzy sets in connections to $\mathcal{C}$-prox.

Keywords: proximity, fuzzy sets, unimodal membership function, thermodynamics, homogeneity, entropy measures.

0 - INTRODUCTION

PROXIMITY (PROX) was introduced (11), to deal with real problems that

1d di not "fit" in metrizable spaces, and even in pseudo-metrics of different kinds were not suitable.

Essentially PROX is a function $\mathcal{R}$ (eventually a generalized one) defined on $X \times X$ and taking values in $\mathcal{G}$, when X is the "formal set" where the images of the real problem are projected and $\mathcal{G}$ is a suitable set.

$\mathcal{R}$ is defined in such a way that a loose triangular law is retained.

Here, the elements of $\mathcal{G}$ are fuzzy sets, with characterizers Γ

/. which are of a special kind, namely unimodal/ the justification for this name is the shape of Γ that is similar to the unimodal distributions.

Proximities based on the axiomatic introduced in Chapter 1 and using unimodal characterizers were named $\mathcal{C}$-Proximities, $\mathcal{C}$-PROX for short.

A topology on X is induced through $\mathcal{R}$.

The object here is only to show that a variety of topologies can be induced in X.

/. /where Finally an application to thermodynamics is presented/ there the concept of homogeneity is a basic one to define measures in the thermostatic space.

Entropy is assimilated to a $\mathcal{C}$-PROX.

Future developments are being pursued in, fundamentally, three directions: Other kinds of $\mathcal{R}$ and $\mathcal{G}$, eventually non-fuzzy, new shapes for Γ i.e., plury-modal characterizers and new applications.

I - Symbology

X is a set.

x , y , z , ω elements of X .

Ω is a lattice.

ε , δ , Σ , $\Gamma(\alpha)$ are elements of Ω .

$\mathbb{R}^+$ non-negative real numbers.

α , β , γ , φ , Ψ are elements of $\mathbb{R}^+$

$\mathcal{G} \in \Omega^{\mathbb{R}^+}$

$\mathfrak{R} : X \times X \to \mathcal{G}$, $(x, y) \mapsto \Gamma_{x,y}$

1 suprémum of lattice

0 infimum of lattice

fe $\vee$ suprímum (or maximum), is a connective in Ω ; sup (max)

$\wedge$ infimum (or minimum), is a connective in Ω ; inf (min).

$\succcurlyeq$ order on Ω .

$\succ$ equivalent to $\succcurlyeq$ and not $=$.

Ax AXIOM

DEF DEFINITION

REM REMARK

TH THEOREM

II - The Axioms

In the sequel X is a set and Ω is a complete distributive lattice, and, in certain cases, an involutive operator $(')$ can be defined such that the order of the lattice is inverted; ($\preccurlyeq$, $\wedge$, $\vee$, 0 and 1) are the order, the connectives inf ($\wedge$) and sup ($\vee$), the infimum and the supremum of the lattice respectively, $\wp$ is a subset of the family of the functions defined over $\mathbb{R}^+ \equiv [0, +\infty)$ with values in Ω , i.e. $\wp \subseteq \Omega^{\mathbb{R}^+}$. At last it is given a function $\mathfrak{R} : X \times X \to \wp, (x, y) \longmapsto \Gamma_{xy}$.

DEF.p0 $\Omega^+ \equiv \Omega \setminus \{0\}$

DEF:p1 A ternary $(X, \Omega, \mathfrak{R})$ is said a $\mathcal{C}$-proximity, if the following axioms are satisfied:

On $\wp$

Sublinhar todo ou apagar em todo.

Ax.1a: If $\Gamma_{xy}(\cancel{\alpha}) \in \wp$, then $\Gamma_{xy}(0) = 0$

DEF.p2: $\Gamma^{x}_{xy} \triangleq \vee\left[\Gamma_{xy}(\alpha), \forall \alpha \in \mathbb{R}^+\right]$

DEF.p3: $\{\alpha^x\} \triangleq \left\{\alpha \in \mathbb{R}^+ \mid \Gamma_{xy}(\alpha) = \Gamma^x_{xy}\right\}$

Ax.1b: $\{\alpha^x\}$ is a closed interval $\left[\varphi^x_{xy}, \psi^x_{xy}\right]$, eventually a singleton, where $\varphi^x_{xy}, \psi^x_{xy} \in \mathbb{R}$ and $\varphi^x_{xy} \leq \psi^x_{xy}$

Ax.1c: If $\Gamma_{xy} \in \wp$, then $\Gamma_{xy}(\alpha_a) \preccurlyeq \Gamma(\alpha_b)$ whenever $0 \leq \alpha_a \leq \alpha_b < \varphi^x_{xy}$ and

$\Gamma_{xy}(\alpha_c) \succcurlyeq \Gamma_{xy}(\alpha_d)$ whenever $\psi^x_{xy} < \alpha_c \leq \alpha_d$,

, $\alpha_a, \alpha_b, \alpha_c, \alpha_d \in \mathbb{R}^+$

Ax.1d: If $\Gamma_{xy} \in \wp$, $\lim_{\alpha \to \infty} \Gamma(\alpha) \to 0$

i.e.

$\forall \varepsilon \in \Omega^+, \cancel{\alpha}, \alpha_0 \in \mathbb{R}^+, \forall \alpha \geq \alpha_0, \Gamma(\alpha) \prec \varepsilon$

Ax.1e: If $\Gamma_{xy} \in \wp$, $\Gamma_{xy}(\cancel{\alpha})$ has, at most, a countable number of discontinuities.

REM/0 The concept of point of discontinuity for monotone functions is the usual one in lattice theory.

DEF.p4: $D_{\Gamma_{xy}} \equiv \{ \alpha \mid \Gamma_{xy}$ is discontinuous at α. $\}$

xx// REM.1: $[\varphi'_{xy}, \psi'_{xy}]$ /will sometimes be represented by/ $\underline{A}(\Gamma_{xy}; = \Gamma^{x}_{xy})$
see REM.6

REM.2: Γ_{xy} is the characterizer or membership function of the fuzzy set just defined, and the shape of Γ_{xy} suggests the name of "unimodal characterizer".

On $\mathcal{R}$

Ax.2a: $\mathcal{R}$ is symmetric, i.e. $\forall x, y \in X, \mathcal{R}(x,y) = \mathcal{R}(y,x) \Leftrightarrow \Gamma_{xy}(\alpha) = \Gamma_{yx}(\alpha), \forall\alpha$

/ $\forall\alpha$. Ax.2b: $\forall x \in X, \mathcal{R}(x,x) \Longleftrightarrow \Gamma_{x,x}(\alpha) = 0 \in \Omega$, /

/. DEF.p5: Let $x, y, z \in X, \alpha, \gamma \in \mathbb{R}^+, \alpha \in [0,\gamma] \subseteq \mathbb{R}^+$ /.

/Let / define $\Gamma_{xyz} : \mathbb{R}^+ \longrightarrow \Omega, \gamma \longmapsto \Gamma_{xyz}(\gamma)$,

and $\Gamma_{xyz}(\gamma) \triangleq V_{\alpha} . \Lambda [\Gamma_{xy}(\alpha), \Gamma_{yz}(\gamma - \alpha)]$

REM.3: This operation is represented by $\oplus$, i.e.

$$\Gamma_{xyz} = \Gamma_{xy} \oplus \Gamma_{yz}$$

DEF.p6: $\Gamma^{x}_{xyz} \triangleq V_{\gamma}[\Gamma_{xyz}(\gamma), \forall\gamma \in \mathbb{R}^+] \in \Omega$

DEF.p7: $\{\gamma^{x}\} \triangleq \{\gamma \in \Omega \mid \Gamma_{xyz}(\gamma) = \Gamma^{x}_{xyz}\}$

Ax.2c: $\{\gamma^{x}\}$ is a closed interval $[\varphi^{x}_{xyz}, \psi^{x}_{xyz}]$, eventually a singleton, where $\varphi^{x}_{xyz}, \psi^{x}_{xyz} \in \mathbb{R}^+$ and $\varphi^{x}_{xyz} \leq \psi^{x}_{xyz}$

Ax.2d: For $x, z \in \Omega$ and $\forall y \in \Omega$, it is

αxxxxx $\Gamma^{x}_{xz} \geq \Gamma^{Y}_{xyz}, \varphi'_{xz} \leq \varphi'_{xyz}$ and $\psi'_{xz} \leq \psi'_{xyz}$

/ if X and Q are topological spaces) Ax.2e: $\mathcal{R}$ is a continuous function /

REM.4: Ax.2d is viewed as fuzzy triangular law and if "crisp" sets and a "crisp" triangular law are used then an "écart" is obtained.

/ REM.5: In some applications, Ax.2b or Ax.2e or both are deleted.

III - Theorems

DEF.p8: $\underline{A}\,(\Gamma_{xy}, \succcurlyeq \mathcal{E}) \triangleq \{\alpha \in \mathbb{R}^+ \mid \Gamma_{xy}(\alpha) \succcurlyeq \mathcal{E}, \mathcal{E} \in \Omega\}$
is the "crisp" set of Γ_{xy}, indexed by $\mathcal{E}$. $\underline{A}$ is a closed interval on $\mathbb{R}^+$.

$$\underline{A}\,(\Gamma_{xy}; \succcurlyeq \mathcal{E}) = [\varphi_{xy \succcurlyeq \mathcal{E}}, \Psi_{xy \succcurlyeq \mathcal{E}}] \subseteq \mathbb{R}^+$$

DEF.p9: $\underline{A}\,(\Gamma_{xy}, \succ \mathcal{E}) = [\varphi_{xy \succ \mathcal{E}}, \Psi_{xy \succ \mathcal{E}}]$
$\triangleq \{\alpha \in \mathbb{R}^+ \mid \Gamma_{xy}(\alpha) \succ \mathcal{E}, \mathcal{E} \in \Omega\} \subseteq \mathbb{R}^+$
where $\succ$ stands for ($\succcurlyeq$ and not $=$). A is the "crisp" set of Γ_{xy}, indexed by $\mathcal{E}$ and an open interval on $\mathbb{R}^+$.

REM.6: referring to REM.1, $\{\alpha^x\}$ can be represented by $\underline{A}\,(\Gamma_{xy}, = \Gamma^x_{xy})$, as the condition imposed is $\Gamma_{xy}(\alpha) = \Gamma^*_{xy}$.

TH.1: $\Gamma_{xyz} = \Gamma_{zyx}$ ⊥ ~~obvious~~

TH.2: $\underline{A}\,(\Gamma_{xy}, \succcurlyeq \mathcal{E}) \equiv \emptyset$, iff $\mathcal{E} \succ \Gamma^x$
identically for $\underline{A}\,(\Gamma_{xy}, \succ \mathcal{E})$

TH.3: If $\underline{A}\,(\Gamma_{xy}, /0)$ is a singleton the characterizer Γ_{xy} is the characterizer of a "crisp" singleton in $\mathbb{R}^x$;

TH.4: For every $x, y, z \in X$, the mapping Γ_{xyz} satisfies the axioms (1a to 1e) if both Γ_{xy} and Γ_{yz} satisfy the same axioms.

PROOF Ax.1a: If $\gamma = 0$ then $\alpha = 0$ and $\gamma - \alpha = 0$ and $\Gamma_{xy}(0) = \Gamma_{yz}(0) = 0 \in \Omega$ and finally $\Gamma_{xyz}(0) = 0 \in \Omega$

PROOF Ax.1b: Let $\mathcal{E}^\circ = \wedge[\Gamma^x_{xy}(\alpha), \Gamma^x_{yz}(\gamma - \alpha)]$, (see DEF.2)

$$\Gamma_{xyz}(\gamma) = \Gamma_{xy}(\alpha) \oplus \Gamma_{yz}(\gamma - \alpha), \qquad \text{(see REM.3)}$$

By DEF.5, 6 and 7 it is $\Gamma^x_{xyz} = \mathcal{E}^\circ$ and $\Gamma_{xyz}(\gamma) = \Gamma^x_{xyz}$ in the closed interval $\underline{A}\,(\Gamma_{xyz}; = \mathcal{E}^\circ) \equiv [\varphi_{xyz = \mathcal{E}^\circ}, \Psi_{xyz = \mathcal{E}^\circ}] \subseteq \mathbb{R}^+$
as $\Gamma^x_{xy} \succcurlyeq \Gamma_{xy}(\alpha)$ and $\Gamma^x_{yz} \succcurlyeq \Gamma_{yz}(\gamma - \alpha)$
and $\Gamma_{xyz}(\gamma) \triangleq \vee \wedge [\Gamma_{xy}(\alpha), \Gamma_{yz}(\gamma - \alpha)]$

$$\Gamma_{xyz}(\gamma) \precsim \mathcal{E}^{o} \quad \text{and} \quad \Gamma^{*}_{xyz} \triangleq \vee\, \Gamma_{xyz}(\gamma)$$

results $\Gamma^{*}_{xyz} = \mathcal{E}^{o}$

PROOF Ax.1c: If $\gamma_1 \leq \gamma_2 \leq \Psi^{*}_{xyz} = \mathcal{E}^{o}$, $\gamma \geq \alpha_2$ and $\gamma \geq \alpha_1$

it is $\Gamma_{xy}(\alpha_2) \succsim \Gamma_{xy}(\alpha_1)$, Ax.1c,

and $\Gamma_{yz}(\gamma - \alpha_1) \succsim \Gamma_{yz}(\gamma - \alpha_2)$, Ax.1c,

then, $\vee \wedge [\, \Gamma_{xy}(\alpha), \Gamma_{yz}(\gamma_2 - \alpha)] \succsim$

$\succsim \vee \wedge [\, \Gamma_{xy}(\alpha), \Gamma_{yz}(\gamma_1 - \alpha)]$,

and $\Gamma_{xyz}(\gamma_2) \succsim \Gamma_{xyz}(\gamma_1)$

similarly:

If $\Psi^{*}_{xyz} = \mathcal{E}^{o} \leq \gamma_3 \leq \gamma_4$

then $\Gamma_{xyz}(\gamma_3) \succsim \Gamma_{xyz}(\gamma_4)$

PROOF Ax.1d: If $\gamma \to \infty$, α and/or $(\gamma - \alpha) \to \infty$

and $\wedge [\, \Gamma_{xy}(\alpha), \Gamma_{yz}(\gamma - \alpha)] \to 0 \in \Omega$

or $\Gamma_{xy}(\alpha \to \infty) \to 0 \in \Omega$

and $\Gamma_{yz}((\gamma - \alpha) \to \infty) \to 0 \in \Omega$

Repetitive

17 REM ~~6~~: $\Gamma_{xyz} \in \Omega^{\mathbb{R}^+}$, by construction.

18 REM. ~~7~~: $\Gamma_{xyx}(\gamma) \neq 0 \in \Omega$, in general and Ax.2d states that $\Gamma^{*}_{xz} \succsim \Gamma^{*}_{xyz}$ but the set γ^{*} (see DEF.p7) may not contain 0 (zero).

PROOF Ax.1e: As both Γ_{xy} and Γ_{yz} have at most countable discontinuities, Γ_{xyz} has too at most countable discontinuities.

19 REM ~~8~~: Γ_{xyz} satisfies Ax.1a to Ax.1e, and the operator $\oplus$ is closed for countable applications and if Γ_{xy} and Γ_{yz}, $\forall x, y, z \in X$, / have no discontinuities for arbitrary applications.

$1\, \Gamma_{xyz}$

IV - Remarks on Topology

As referred in the introduction, the main goal here is to point out the adaptability of the $\mathcal{C}$ -prox..

The topology endowed in X depends up on each application. A natural suggestion is that the topology in X be such that the mapping $\mathcal{R} : X \times X \to \Omega^{\mathbb{R}^+}$ be continuous, when $\Omega^{\mathbb{R}^+}$ is endowed with a topology adjusted to the application.

It is remembered that the concept of discontinuity of Γ_{xy} supposes already that some kind of topology is in Ω (see REM.0). Any finer topology than the initial one is suitable. Work is being done now on the concept of fuzzy balls to endow in X with a suitable topology.

Many authors have suggested topologies for Ω or $\Omega^{\mathbb{R}^+}$, e.g. See Ref: 1, [illegible] 3, 5, 7, 8, 9, 10, 11, 18.

V - APPLICATION OF PROXIMITIES TO THERMODYNAMICS

A) Homogeneity

// J. $\mathcal{T}$ is an universal class of sets T J.

J. $\Pi_\gamma(T)$ is a partition of T

J. $T_\alpha, T_\beta \in \Pi_\gamma(T)$

φ is a set of linearly independant real measures (or σ-measures)

real numbers (1) ..... $\mu_i : T \longrightarrow R$ where: $\mu_i \in \varphi$, $T \in \mathcal{T}$ and R (Reals) ?

$\Pi_\gamma(T)$ being a partition of T, the following expressions apply:

(2) ..... $T_\alpha \cap T_\beta \equiv \phi \, , \forall \, (\alpha \neq \beta)$

(3) ..... $\bigcup\limits_\alpha (T_\alpha) = T$

DEF. 1 Partition $\Pi_\gamma(T)$ is φ-homogeneous if:

$$(4) \;..... \quad \frac{\mu_i(T_\alpha)}{\mu_i(T_\beta)} = \frac{\mu_j(T_\alpha)}{\mu_j(T_\beta)} \, , \quad \begin{array}{l} \forall \, T_\alpha, T_\beta \in \Pi_\gamma(T) \\ \forall \, \mu_i, \mu_j \in \varphi \end{array} \quad \text{and}$$

expression (4) can be easily transformed in (5):

$$(5) \quad \frac{\mu_i(T_\alpha)}{\mu_i(T)} = \frac{\mu_j(T_\alpha)}{\mu_j(T)} \, , \quad \begin{array}{l} \forall \, T_\alpha \in \Pi_\gamma(T) \\ \forall \, \mu_i, \mu_j \in \varphi \end{array}$$

DEF. 2 The fineness ρ of a partition $\Pi_\gamma(T)$ is defined by (6):

$$(6) \quad \mathrm{Max} \left[\frac{\mu_i(T_\alpha)}{\mu_i(T)} \, , \; \forall \, T_\alpha \in \Pi_\gamma(T) \right] = \rho_{\Pi_\gamma}(T)$$

Taking in consideration (4) and (5), $\rho_{\Pi_\gamma}(T)$ is independant of μ_i.

DEF. 3 : $\Pi_T \equiv \left\{ \pi_\gamma(T) : \wp_{\pi_\gamma(T)} \leq \wp \right\}$ ..... (7)

is the set of all partitions of T that have finenesses less than $\wp$.

Π_T is $\wp\varphi$-homogeneous.

The set T possessing a non void set of partitions $\wp\varphi$-homogeneous is defined as $\wp\varphi$-homogeneous.

DEF. 4: $\mathcal{G}^x \equiv \left\{ T : T \in \mathcal{G} \text{ and } T \ \wp\varphi\text{-homogeneous} \right\}$.

$\mathcal{G}^x$ is a sub-set of $\mathcal{G}$ and is considered $\wp\varphi$-homogeneous.

DEF. 5: If:

$$\forall \mu_i \in \varphi \ , \ \mu_i(T) = \mu_i(T') \Longleftrightarrow T \equiv T'$$

$$\mu_i \in \varphi \ , \ \mu_i(T) \neq \mu_i(T') \Longleftrightarrow T \not\equiv T'$$

for $\forall$, T, $T' \in \mathcal{G}^x \subseteq \mathcal{G}$ and $\wp\varphi$-homogeneous.

Then φ is an "adequate" set of linearly independant real measures (σ-measures) for $\mathcal{G}^x$.

Note that any other real measures on T can be expressed as a linear homogeneous function of degree 1 of the measures belonging to φ.

B) An Axiomatic for Thermostatic

Ax. 1 : All thermodynamic system is an universal class of sets $\mathcal{G}^x$, $\wp\varphi$-homogeneous and φ is an "adequate" set of linearly independant real measures (σ-measures) for $\mathcal{G}^x$.

Card $\varphi = N$, finite.

For $\wp \longrightarrow 0$ (zero), $\forall \mu_i \in \varphi$. $\mu_i(T)$ is continuous on $\mathcal{G}^x$.

Ax. 2 : There are two real measures (σ-measures), entropy μ_s and internal energy μ_u.

If $\varphi \equiv \left\{ \mu_s ; \mu_i \cdots \mu_j \right\}$ then $\mu_u = F[\varphi]$ and $\varphi \cup \mu_u$ is a N + 1 Euclidean Convex Space.

μ_u and μ_s are dual functions, exgratis:

$$(\min \mu_u)_{\mu_s = \text{const.}} = (\max \mu_s)_{\mu_u = \text{const.}}$$

Note 1 : Continuous trajectories (lines) can be described on the surface

$$\mu_u - F\left[\mu_s , \mu_i \ldots , \mu_j \right] = 0 \text{ if } \wp \rightarrow 0.$$

Note 2 : The thermodynamic space is not metrisable, but a proximity can be defined, as it will be shown in C).

C)Proximity in thermodynamics

1) Reversible and irreversible trajectories

In all trajectories (reversible or otherwise) the starting state T^x and finishing state T^y belong to $\mathcal{G}^x$ $\wp\varphi$-homogeneous.

If all the other intermediate states belong to $\mathcal{G}^x$ then the trajectory is declared reversible, if not irreversible.

A general irreversible trajectory is symbolised in the following fashion:

$T^x \nearrow\!\!\searrow T^y$, T^x, T^y being respectively the starting and finishing point and $T^x, T^y \in \mathcal{G}^x$.

Reversible trajectories are represented as follows: $T^x \longrightarrow T^y$

2) Definition of a Proximity on $\mathcal{G}^x$

$$\mathcal{H} = \bigtimes_i \mu_i , \forall \mu_i \in \varphi \quad \text{(Cartesean Product)}$$

The proximity of two states T^x T^y is given by:

$$\mathcal{H}(x,y) = \Gamma_{xy}(\alpha)$$

where $\Gamma_{x,y}(\alpha) \in \mathcal{G}^T \subseteq \mathcal{G}$ (defined in 2 e)

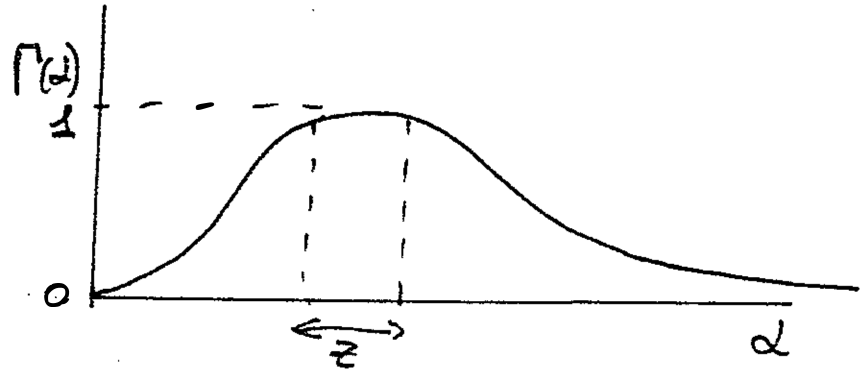
$\mathcal{G}^T$ satisfies to the following conditions:

a) $\mathcal{G}^T \in \mathcal{G}$

b) $\Gamma(0) = 0$

c) The non-decreasing branch starts at $\alpha = 0$.

Thus the general aspect of Γ is the following:



z corresponds to the region of the "real" trajectories, the most plausible, and the reversible trajectories to $\alpha = 0$, $\Gamma(0) = 0$, which can be interpreted as "impractible" because $\Gamma(0) = 0$.

Comments:

1) All reversible trajectories are equiproximate $\alpha = 0$, and their likelihood, $\Gamma(\alpha = 0) = 0$, is zero, physically ideal.
2) All irreversible trajectories correspond to $\alpha > 0$ and their likelihood is positive $\Gamma(\alpha > 0) > 0$.
3) The most likely proximity corresponds to the zone where $\Gamma(\alpha)$ is maximum.
4) Considering the two trajectories, $T^x \frown T^y \frown T^z \Longleftrightarrow \Gamma_{xyz}(\gamma)$ and $T^x \frown T^z \Longleftrightarrow \Gamma_{xz}(\gamma)$. The zone of higher likelihood is shifted to greater values of α in Γ_{xyz} than in Γ_{xz} .

D) Entropy Production

If α is interpreted as entropy production, $\alpha \equiv \delta S$, then $\Gamma_{xy}(\alpha) \equiv \Gamma_{xy}(\delta S)$ and some simple conclusions can be taken:

- In a reversible trajectory (process), $\alpha = 0$ then $\Gamma_{xy}(0) = 0$, the likelihood of such a process is nill.
- The max $[\Gamma_{xy}(\alpha)] = \Gamma^*$ corresponds to the entropy production $\delta S = \alpha$ more likely to occur.
- The set $\{\alpha : \Gamma_{xy}(\alpha) \geqslant \delta \leqslant \Gamma^*\} \equiv [\delta_a, \delta_b]$ is an interval of occurence of trajectories which are δ-likely to occur.

- If, $\max\left[\Gamma_{xyz}(\gamma)\right] = \Gamma^{*}_{xyz}$ and $\max\left[\Gamma_{xz}(\alpha)\right] = \Gamma^{*}_{xz}$

and

$$[\gamma^{*}_{a}, \gamma^{*}_{b}] \equiv \{\gamma : \Gamma_{xyz}(\gamma) = \Gamma^{*}_{xyz}\}$$
$$[\delta^{*}_{a}, \delta^{*}_{b}] \equiv \{\alpha : \Gamma_{xz}(\alpha) = \Gamma^{*}_{xz}\}$$

then $\Gamma^{*}_{xz} \geq \Gamma^{*}_{xyz}$

and $\delta^{*}_{a} \leq \gamma^{*}_{a}$ and $\delta^{*}_{b} \leq \gamma^{*}_{b}$

which means the entropy production in the process $T^{x} \nearrow\!\searrow T^{y} \nearrow\!\searrow T^{z}$ is greater than in the process $T^{x} \nearrow\!\searrow T^{z}$, for the same likelihood (or level of membership).

E) Time and Entropy Production

If time t is considered a monotonous function of $1/\alpha \equiv 1/\delta S$, some interesting interpretations are possible.

a) If $\alpha = 0$ then $t = \infty$. A process that would take ∞ time for completion would be eventually reversible.

b) The typical t^{*} (or the most likely time) would correspond to $\Gamma^{*}\left(\max \Gamma(\alpha)\right)$.

c) As $\alpha \to \infty$, $t \to 0$, and $\Gamma(\alpha) \to 0$. this could be interpreted as follows: when the time of the process is less than t^{*}, then the likelihood of the process would diminish tending to zero with $t \to 0$.

Conclusion

Space $\mathfrak{X} \equiv \varphi$ can be topologically structured with a class $\mathcal{G}' \leq \mathcal{G}$ of proximities and some form of a fuzzy distance. Proximity, between thermodynamic states can be defined.

Entropy production is a monotonous function of α, eventually $\alpha \equiv \delta S$.

Time is an inverse function of δS.

V - Conclusion

The flexibility of Proximities gives a certain amount of freedom in the structuring of X , by choosing a convenient $\mathcal{R} : X \rightarrow B \in \Omega^{\mathbb{R}^+}$.

$\mathbb{R}^+$ can be substituted by a suitable compact, various Ω and respective topologies are presented in the literature and can be used.

In some applications to graphs and hyper-graphs, one can delect Ax.2b and when the real system projected in X is non-homogeneous, Ax.2c can be supressed.

Work is being done on proposing suitable topologies for Ω and X .

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